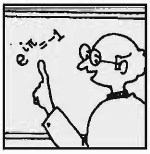

In this section of* Resonance, *we invite readers to pose questions likely to be raised in a classroom situation. We may suggest strategies for dealing with them, or invite responses, or both. "Classroom" is equally a forum for raising broader issues and sharing personal experiences and viewpoints on matters related to teaching and learning science.

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Rayleigh – Benard Convection
Physics of a Widespread Phenomenon

Introduction

Convection should be familiar to people who have observed a lighted candle, heated broth and the shimmering of air currents over a paved road on a hot day. It is responsible for the transfer of heat from the core to the surface in a main sequence star like our sun, the flow of water in the water-wall-panels of the steam generator of a thermal power plant, the winds over the earth, the currents in the oceans, the fact that freezer cabinets are always kept on the top of a household refrigerator!

Convection refers to heat transfer processes effected by the flow of fluids. The very word has its roots in the Latin verbs *convecto-are* and *conveho-vehere* which means to bring together or to carry into one place. Understanding convection requires a sound knowledge of heat transfer and fluid mechanics as the theory of convection rests on both these subjects.

Two types of convection mechanisms are distinguished – free or natural and forced. In forced convection an externally induced relative motion between the convective system and the surroundings prevails. In free convection the relative motion is due only to the convective instability itself. We analyze free or natural convection in this article.

The Nature of the Problem

The most fascinating thing about convection is that even the simplest system undergoing convective motion cannot yet be given an exact analytical mathematical description. The partial differential equation(s) that describe the convective flow analytically have been studied for the past 200 years – with rewarding results – but the exact analytical solutions of these are yet to be found!

A fluid layer heated from below, a supposedly simple system showing convection, experiences forces that drive the convective flow, resulting from the buoyancy of the heated layer. The magnitude of such forces depends on the temperature difference prevailing between the top and bottom portion of the fluid layer.

Early Theories

An early description of convection was given in the 1790s by Benjamin Thompson, Count Rumford – which he used to account for the transfer of heat in an apple-pie! Only in the 1900s were systematic investigations undertaken.

The most significant and pertinent experimental work was carried out by the Frenchman Henri Benard. He studied a seemingly simple convective system which he never knew was so complicated that the real physics behind it was uncovered only much later!

In the 1900s convection was one of the myriad things that John William Strutt, Lord Rayleigh studied in his illustrious and prolific career(see *Box* 1). Rayleigh, according to Chandrasekhar, had said the last word on many subjects and sealed them once and for all! In one of his last articles, published in 1916, he attempted to explain what is now known as *Rayleigh–Benard convection.* His work remains the starting point for most of the modern theories of convection. This article explains Rayleigh's theory.

Box 1

Lord Rayleigh was a British physicist born near Maldon, Essex, on November 12, 1842. He was educated at Trinity College, Cambridge, where he graduated as Senior Wrangler in 1865. As a successor to James Clerk Maxwell, he was head of the Cavendish Laboratory at Cambridge from 1879–1884, and in 1887 became Professor of Natural Philosophy at the Royal Institution of Great Britain. He was elected a Fellow of the Royal Society in 1873 and served as its president from 1905–1908. He received the Nobel Prize for Physics in 1904 for his 1894 collaborative discovery (with Sir William Ramsay) of the inert elementary gas argon.

Rayleigh's research covered almost the entire field of physics, including sound, wave theory, optics, colour vision, electromagnetism, the scattering of light, hydrodynamics, capillarity, viscosity, the density of gases, photography and elasticity, as well as electrical measurements and standards. His research on sound was embodied in his 'Theory of Sound' and his other extensive studies in physics appeared in his 'Scientific Papers'. Rayleigh died on June 30, 1919 at Witham, Essex.

Rayleigh's Theory

Model: A fluid with simplified properties, as against a real one, is considered in the two dimensional model to be constructed to explain convection. A thin layer of the fluid is confined fully between two semi-infinite flat plates so that there is no gap. By a thin layer here we again mean that the horizontal dimension of the fluid layer is very large when compared to the vertical. This ensures that the influence of the side walls is minimized.

The fluid has to be heated in such a way that the temperature gradient in the fluid remains uniform (spatial invariance) and steady (temporal invariance). This, in other words, simply means that the graph of temperature vs height is a straight line when the fluid is at rest.

Assumptions

(1) **The fluid is incompressible** – which is valid as the layer is shallow.

(2) The **density** of the fluid is the only property that gets affected by the change in the temperature across it.

(3) It experiences **an uniform gravitational force** over its entire volume.

In this model consider a packet of fluid displaced by a small amount above or below its present position. For understanding what is happening in the model one has to be familiar with some basic concepts: buoyancy, viscosity and thermal diffusivity of a a fluid.

Buoyancy

The pressure in the water increases as we go down because the weight of the fluid layer above each point contributes to the pressure experienced at that point.

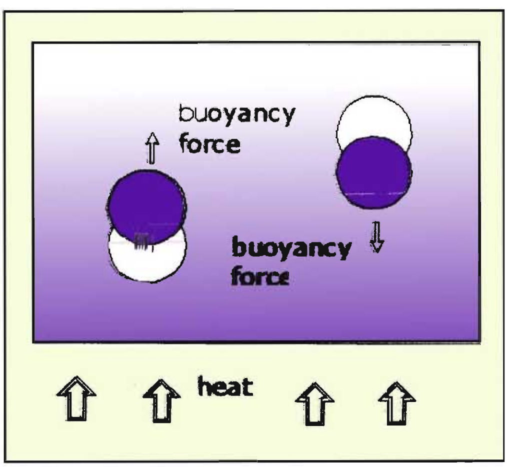
Without a temperature gradient buoyancy effects will be due to the pressure difference across the fluid packet which are balanced by the weight of the packet. This ensures static equilibrium.

With heating, the fluid packet considered has a lower density at the bottom relative to the surroundings which makes it rise owing to the increase in buoyancy force which disrupts the static equilibrium.

Elementary Explanation

Consider a fluid packet at the bottom of the trough with heat supplied to it from below, as shown in *Figure* 1. This packet has a higher temperature and so has a lower density when compared to the average density of the entire layer. A similar packet at the top will have relatively higher density due to its lower temperature.

Figure 1.



Suppose now due to some random fluctuation a displacement is given to the packet in the upward direction. This will result in an imbalance in the forces acting on the packet. The packet is now pushed up into a region of higher density. This creates a positive buoyancy as explained above which causes the packet to rise further.

The upward force is proportional to the density difference and volume of the packet. As the fluid packet raises through regions of relatively colder fluid whose average density progressively increases, it results in an increased density difference between the packet and the surroundings which accelerates the rise.

Similarly a downward push on a packet of fluid makes it enter a region of lower average density thus propelling it down. It would sink and the initial disturbance is enhanced. Thus the whole of the fluid layer would tend to overturn resulting in an exchange of the fluid between the hot and cold ends.

When the fluid tries to move, or circulate, it initially does so with a small velocity. When the fluid packet moves, its motion is impeded by the 'viscous drag' between it and the surrounding fluid.

It seems from this analysis that convection will be observed in a fluid region whenever there is a temperature gradient, however small it may be. But such sensitive dependence of the initiation of the flow on the temperature gradient is not observed in actual circumstances. There seems to be a minimum value, above which convective flow results. This was, characteristically, explained by Rayleigh.

Rayleigh Number and its Physical Significance

The onset of convection has to take into consideration two more modes of energy dissipation in the fluid. In other words, the force imbalance equation which explains the convective motion has to be recast to accommodate two more effects.

One of our initial assumptions is that before the temperature gradient prevails the fluid is at rest and is not subjected to any external influence which might induce motion. So when the fluid tries to move, or circulate, it initially does so with a small velocity. When the fluid packet moves, its motion is impeded by the 'viscous drag' between it and the surrounding fluid.

Viscosity, as we know, is internal fluid resistance offered to a layer of fluid sliding on another. It is given by the formula

$$T = \mu \mathrm{d}u/\mathrm{d}y$$

where T = the shear stress applied, μ = the dynamic viscosity and du/dy = the change in the velocity component in a perpendicular direction. It is a frictional force acting in the opposite direction to relative motion. In our fluid packet, this acts against the buoyancy force and tries to impede the motion.

The second dissipative effect is from the fact that convection is not the only mode of heat transfer that could happen in the given circumstance – transfer by heat conduction cannot be ignored.

In reality, when the fluid packet is displaced into cooler surroundings due to the buoyancy force, this heat energy is conducted into the surrounding fluid as a temperature difference prevails. For the fluid packet coming down from a cooler environment the transfer is in the other way leading to similar results. Since the local temperature difference is reduced by heat diffusion it results in a reduction in the buoyancy force.

Convection sets in when the Rayleigh number exceeds a certain critical value.

It is necessary that the buoyancy force, which is the result of the internally applied temperature gradient, must exceed the dissipative forces of viscous drag and heat diffusion to ensure the onset of convective flow.

To understand better the influence of these forces on the onset of convection, we use a non-dimensional number *Ra*, called the Rayleigh number, which is proportional to buoyant force divided by the product of the viscous drag and the rate of heat diffusion.

$$Ra = g\beta\Delta T L^3/\alpha\nu$$

where, β is the coefficient of thermal expansion, ΔT the temperature difference between hot and cold ends, L the fluid thickness, α the thermal diffusivity and ν the kinematic viscosity[1] of the fluid. Convection sets in when the Rayleigh number exceeds a certain critical value.

Stability Analysis

As convection is due to force imbalance, it is often convenient to

[1]Kinematic viscosity is the ratio of $-\mu$ to the density ρ of the fluid.

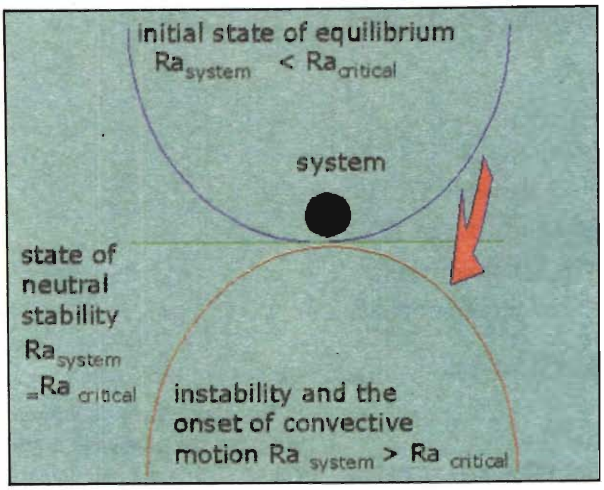
Experiments reveal a critical Rayleigh number of 1708 above which convective instability begins.

analyze it in terms of stability. This analysis is given in *Figure* 3.

A system is usually found in the minimum energy state of a potential surface, which is what the lowest point in the bowl in the figure represents. An increase in the Rayleigh number beyond the critical value takes the system from this point of maximum stability to the point of maximum instability of the system resulting in the onset of a convective motion. By continuity we can predict that for a particular value of Rayleigh number, the potential surface will be a straight line of neutral stability (see *Figure* 2). Experiments for the geometry considered reveal a critical Rayleigh number of 1708 above which convective instability begins.

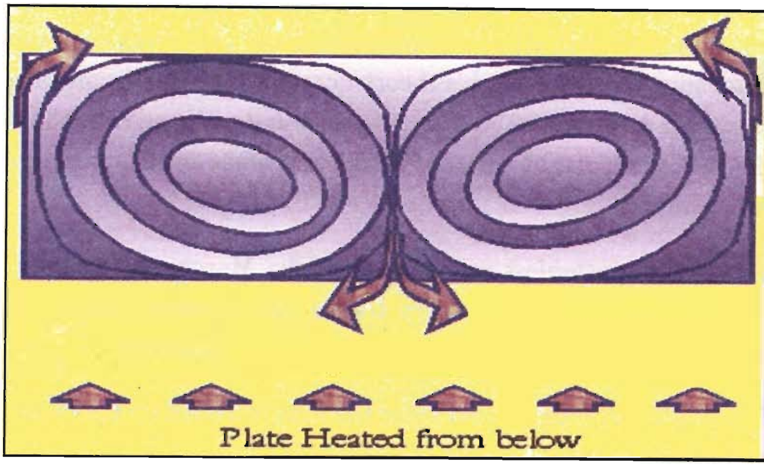
Rayleigh's theory of the onset of convective motion in a fluid layer enclosed between two plates, inspite of many simplifying assumptions, nevertheless, explains successfully the conditions required for the initiation of convection in real fluids. But even the more comprehensive theories of later years cannot explain all the observed features of a fully developed convective flow in enclosed spaces. Only qualitative descriptions are possible.

Figure 2. Classical marble-in-a-bowl analogy of stability analysis of onset convective flow.



Convection Cells

When the critical Rayleigh number is exceeded and as instability sets in, the hot layer tries to go up simultaneously with the cold upper layer coming down. Both things cannot happen at the same place and at the same time. The fluid avoids this stalemate by separating itself into a pattern of convective cells. In each cell the fluid flows in a closed loop and the direction of flow alternates with successive cells. This roll when viewed in cross-section resembles a bloated square the height of which is determined by the width of the fluid layer.



Energy is transferred from bottom to top by the convection cells formed in the fluid when Ra exceeds 1708. The direction of rotation of two adjacent cells are shown in the figure.

Figure 3. Rayleigh–Benard convection cells (computer model).

Figure 3 represents the Rayleigh–Benard convection cells formed inbetween horizontal plates when convection sets in.

The Convective Engine

A free convective flow superficially resembles a heat engine – a contrivance which by energy interaction with two external heat reservoirs gives useful work output. For example we could imagine a propeller being rotated by the flow in a convective cell. In practice, this useful work is small compared to the maximum allowed by the temperature difference. The reason for the non-ideal behaviour is clear – this engine is far from reversible because of viscosity and heat conduction.

Modifications of the Rayleigh Theory

Lord Rayleigh's analysis of the problem of convective flow was initiated by the experiments of Benard. This theory unfortunately assumes an ideal experiment which is in a subtle way different from the actual experiments of Benard. So this theory fails in explaining the details of Benard's experiments.

The experimental conditions Benard employed were different in the sense that the fluid layer is not just confined between two horizontal rigid plates, as assumed in the model above, but is open to air at the upper surface. Since the surface is free, surface tension forces can affect the flow, which can overcome even the buoyancy force. Rayleigh's theory does not take this factor into account. A successful alternate theory was developed in 1958 by J R A Pearson of the Imperial College of Science and Technology in London.

A new dimensionless number called the Marangoni number, named after the 19th century Italian investigator, enters the theory if the effects of surface-tension are considered.

The Rayleigh's theory and other theories modeled on it explain the conditions required for the onset of convective flow but fail to explain what happens once the flow is initiated.

Suggested Reading

[1] M G Velarde and C Normand, Convection, *Scientific American*, 1980.

1877

The editors of *Scientific American* who have just witnessed a remarkable demonstration of new technology in their offices, recall the event for readers:

"Mr Thomas A Edison recently came into this office, placed a little machine on our desk, turned a crank, and the machine enquired as to our health, asked how we liked the phonograph, informed us that it was very well, and bid us a cordial good night."

Scientific American, September 1995

Errata

(Vol.4, Number 4, April 1999, page 89)

1. In the boxed item, the maximum number of electrons per unit area in the lowest energy state (degeneracy factor) is indicated as hB/q should be replaced with qB/h.

2. The parenthetical statement, in the context of FQHE observations of Tsui and Stormer: (for a lower magnetic field) should be replaced with (for a higher magnetic field).